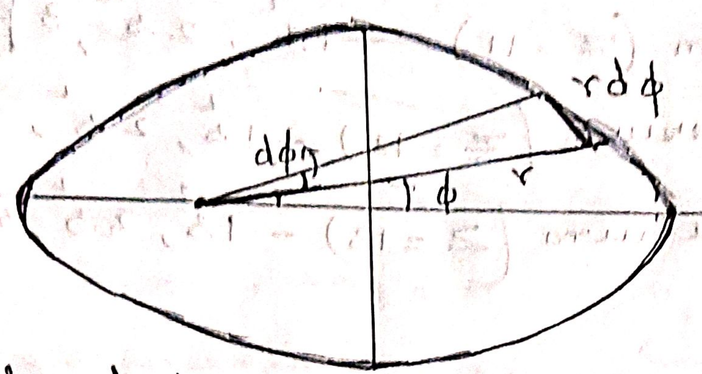
lim of elements.

(a) Magnetic dipole moment due to orbital motion of the electron.

Consider an electron moving in an elliptical orbit of area A with a period T.



The electron crosses any point in the orbit $1/T$ times in unit time. This is equivalent to a current $i = e/T$ in a loop of area A, where e is the charge of the electron. Applying Ampere's theorem, this current gives rise to a magnetic dipole moment μ_i, given by

$$\mu_i = iA$$

$$\mu_i = \frac{e}{T} A \quad \text{——— (1)}$$

Since the areal velocity in a central orbit is $\frac{1}{2} r^2 \frac{d\phi}{dt}$,

the area

$$A = \int_0^r \frac{1}{2} r^2 \left(\frac{d\phi}{dt}\right) dt$$

Now, the angular momentum of the electron

$$L = m r^2 \frac{d\phi}{dt} = \text{constant}$$

or $\frac{1}{2} r^2 \frac{d\phi}{dt} = \frac{L}{2m}$ = constant

$$A = \int_0^T \left(\frac{L}{2m}\right) dt$$

$$A = \frac{LT}{2m} \quad \text{--- (2)}$$

Sub (2) in (1)

$$\mu_l = \frac{e}{2m} L \quad \text{--- (3)}$$

since angular momentum is quantised, we have $L = \frac{l h}{2\pi}$

$$\therefore \mu_l = \frac{e}{2m} \cdot \frac{l h}{2\pi} \Rightarrow \left(\frac{e h}{4\pi m}\right) l \quad \text{--- (4)}$$

hence the orbital magnetic dipole moment of the electron is directly proportional to the orbital quantum number l; ie the magnetic dipole moment of the orbital electron is therefore, an integral multiple of the quantity $\frac{eh}{4\pi m}$. $\frac{eh}{4\pi m}$ is the smallest unit of magnetic dipole moment and is called the Bohr electron magneton. It serves as a natural atomic unit of magnetic moment. It is represented by the symbol μ_B.

$$\mu_B = \frac{eh}{4\pi m} = \frac{(1.602 \times 10^{-19})(6.626 \times 10^{-34})}{4\pi (9.109 \times 10^{-31})}$$

$$= 9.274 \times 10^{-24} \ J (wb/m^2)^{-1}.$$

(b) Magnetic dipole moment due to spin

An electron spinning about its axis should also behave as a tiny magnet and possess a magnetic dipole moment due to this spin. However, nothing is known about the shape of an electron or the manner in which its charge is disturbed. Hence it is impossible to calculate its spin magnetic dipole moment in a manner analogous to that used for the orbital motion. In order to obtain agreement with experiment results, the spin magnetic dipole moment (μ_s) is assigned the value

$$\mu_s = 2 \times \frac{e}{2m} S \quad \text{where } S = \frac{sh}{2\pi}$$

$$\therefore \mu_s = 2 \frac{e}{2m} \frac{sh}{2\pi} = \left(\frac{2eh}{4\pi m}\right) s$$

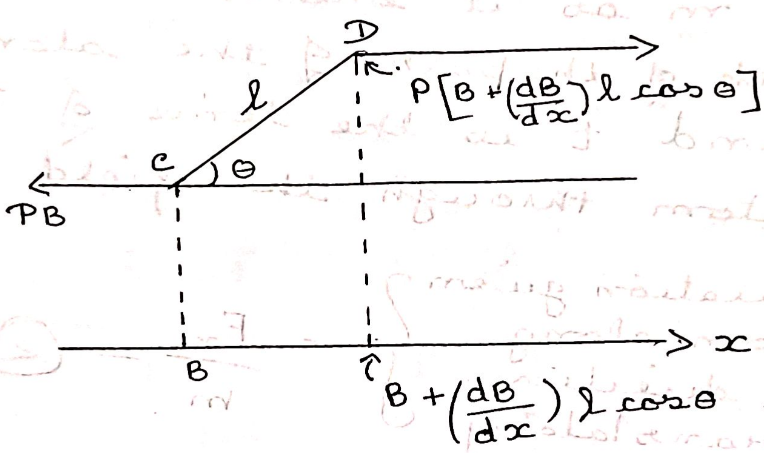
if $s = 1/2$ then

$$\mu_s = \frac{eh}{4\pi m}.$$

* The stern and Gerlach experiment.

Principle.

The experiment is based on the behaviour of a magnetic dipole (atomic magnet) in a non-uniform magnetic field. In a uniform magnetic field (B) the dipole experiences a torque that tends to align the dipole parallel to the field. If the dipole moves in such a field in a direction normal to the field, it will trace a straight line path without any deviation. In an inhomogenous magnetic field, the dipole experiences, in addition, a translatory force. If the such an atomic magnet flies across such an inhomogenous magnetic field normal to the field direction, it will be deviated away from its rectilinear path.



Let the magnetic field vary along the x-direction, so that the field gradient is $\frac{dB}{dx}$ and is positive.

CD is the atomic magnet (of pole strength P, dipole moment M) with its axis length l, inclined at an angle θ to the field direction. If the field strength at the pole C is B, then the field strength at the other pole D will be $B + \frac{dB}{dx} l\cos\theta$.

Hence the forces on the two poles are PB and $P\left[B + \frac{dB}{dx} l\cos\theta\right]$. Hence the atomic magnet experiences not only a torque ($= PlB = \mu_s B$) but also a translatory force.

$$F_x = \frac{dB}{dx} \cos\theta \, Pl$$

$$\therefore F_x = \frac{dB}{dx} \mu_s \cos\theta \quad \text{— (1)}$$

Let v = velocity of the atomic magnet of mass m as it enters the field.

L - length of the path of the atom in the field and t is the time of travel of the atom through the field $= \frac{L}{v}$

The acceleration given to the atom along the field direction by the translatory force F_x $\Big\} = \frac{F_x}{m}$ — (2)

$\left[\frac{vel}{} \frac{dist}{ti} \quad ti = \frac{dist}{vel} \right.$

$\left(F = ma \quad \frac{F}{m} = a \right)$

The displacement of the atom along the field direction, on emerging out of the field. $\Big\} = d$

$S = \frac{1}{2}at^2$

$$= \frac{1}{2}\left(\frac{F_x}{m}\right) t^2$$

$$= \frac{1}{2} \frac{F_x}{m} \frac{L^2}{v^2}$$

$$= \frac{1}{2}\left(\frac{\frac{dB}{dx}\mu_s \cos\theta}{m}\right) \cdot \frac{L^2}{v^2} \quad \text{— (3)}$$

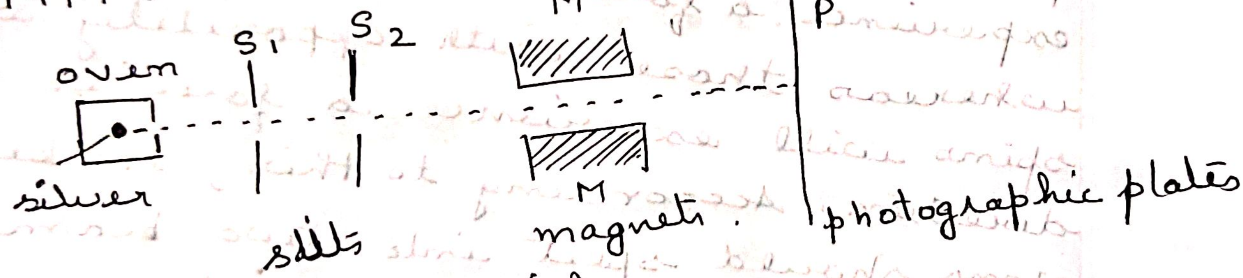
If μ is resolved component $\parallel$ to the magnetic moment in the field direction

$$\mu = \mu_s \cos\theta$$

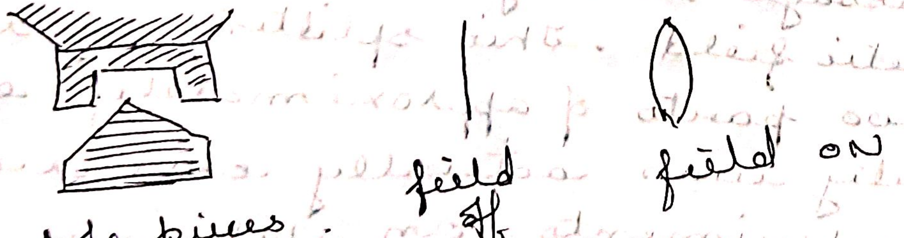
$$\alpha = \frac{1}{2}\frac{dB}{dx}\frac{\mu}{m}\frac{L^2}{v^2} \quad \text{— (4)}$$

Experimental arrangement:

Silver is boiled in an oven. Atoms of silver stream out from an opening in the oven. By the use of slits S_1 and S_2, a sharp beam of atoms is obtained. These atoms then pass through a very inhomogeneous magnetic field between the shaped poles of a magnet MM.



(a)



(b)

A high degree of non-uniformity in the magnetic field is produced by making one of the pole pieces of a powerful electromagnet a knife edge and the other flat with a groove cut

cut in it opposite to knife edges.

(c)

The lines of forces are close together at the knife edges and the field there is much stronger than that at the other pole pieces. The magnetic field is at right angles to the direction of movement of the atoms. Finally the atoms fall on a photographic plate P. The whole arrangement is enclosed in an evacuated chamber.

With no field, the beam produces a narrow continuous line on the plate (fig C). In terms of vector atom model, those atoms, with electron spins directed parallel to the magnetic field, will experience a force in one direction, whereas those with oppositely directed spins will experience a force in opposite direction. According to this, the beam of atoms should split into two beams in its passage through the inhomogeneous magnetic field. This splitting of the beam into two parts of approximately equal intensity was actually observed in these experiments. On applying the inhomogenous magnetic field, it was found that the stream of silver atoms split into two separate lines. Knowing $\frac{dB}{dx}$, L, V and d, μ was calculated. It was found that each silver atom had a magnetic moment of one Bohr magneton in the direction of the field.